Spacecraft to spacecraft coherent laser tracking as a xylophone interferometer detector of gravitational radiation

Massimo Tinto

Jet Propulsion Laboratory, California Institute of Technology
Pasadena, California 91109

Abstract

Searches for gravitational radiation can be performed in space with two spacecraft tracking each others with coherent laser light. Four observable can be measured: two one-way Doppler (each spacecraft transmits a light beam and the other reads out the Doppler with an onboard laser), and two two-way Doppler (light is coherently transponded by the other spacecraft and Doppler is extracted by comparing the phases of transmitted and transponded lights). One-way and two-way tracking data recorded on board the two spacecraft are time tagged and telemetered back to Earth for data analysis. By linearly combining the four data sets, we derive a method for reducing by several orders of magnitude, at selected Fourier components, the frequency fluctuations due to the lasers. The gravitational wave signal remaining at these frequencies makes this spacecraft to spacecraft coherent laser tracking technique the equivalent of a *xylophone interferometer* detector of gravitational radiation. Estimates for the strain sensitivities achievable with these experiments are presented for gravitational wave bursts, monochromatic signals, and a stochastic background of gravitational radiation. This experimental technique could be implemented with two spacecraft carrying an appropriate optical payload, or with the proposed broad-band, space-based laser interferometer detectors of gravitational waves operated in this non-interferometric mode.

PACS numbers: 04.80.N, 95.55.Y, and 07.60.L

I Introduction

The detection of gravitational radiation is one of the most challenging efforts in the physics in this century. A successful observation will not only represent a great triumph in experimental physics, but will also provide a new observational tool for obtaining a better and deeper understanding about its sources, as well as a unique test of the proposed relativistic theories of gravity [1].

Over the past forty years several designs of Earth-based as well as space-based detectors have been considered in the form of feasibility studies, prototypes, or fully operational instruments. Earth-based detectors, such as resonant bars and laser interferometers, are most sensitive to gravitational waves in the frequency interval 10 Hz to about 10 kHz. The low frequency limit is imposed by the seismic and gravity-gradient noise, while the high frequency cut-off is determined by instrumental noise sources [1]. Space-based detectors instead, such as the coherent microwave tracking of interplanetary spacecraft [2] and proposed laser interferometers in planetary orbits [3], are most sensitive to a complementary frequency band, from about 10^{-5} Hz to 1 Hz.

In present single-spacecraft Doppler tracking experiments in particular, many of the noise sources can be either reduced or calibrated by implementing appropriate microwave frequency links and by using specialized electronics. The fundamental instrumental limitation is imposed by the frequency (time-keeping) fluctuations inherent to the reference clocks that control the microwave system [4, 5, 6]. Hydrogen maser clocks, currently used in Doppler tracking experiments, achieve their best performance at about 1000 seconds integration time, with a fractional frequency stability of a few parts in 10^{-16} [4]. This is the reason why these interferometers in space are most sensitive to millihertz gravitational waves. This integration time is also comparable to the propagation time to spacecraft in the outer solar system. The frequency fluctuations induced by the intervening media have severely limited the sensitivities of these experiments. Among all the propagation noise sources, the troposphere is the largest and the hardest to calibrate to a reasonably low level. Its raw frequency fluctuations have been estimated to be as large as a few parts in 10^{-14} at 1000 seconds integration time [4].

In order to remove systematically the effects of the troposphere in the Doppler data, it was shown by Vessot and Levine [7] and Smarr *et al.* [8] that by adding to the spacecraft payload a highly stable frequency standard, a Doppler read-out system, and by utilizing a transponder at the ground antenna, one could make

Doppler one-way (Earth-to-spacecraft, spacecraft-to-Earth) as well as two-way (spacecraft-Earth-spacecraft, Earth-spacecraft-Earth) measurements. This configuration makes the Doppler link totally symmetric and allows, by properly combining the Doppler data recorded on the ground with the data measured on the spacecraft, the complete removal of the frequency fluctuations due to the Earth troposphere, ionosphere, and mechanical vibrations of the ground antenna from the new formed data.

In our previous paper [9] (which from now on we will refer to as Paper 1) we have shown that the results obtained by Vessot and Levine could be further improved. In Paper 1 we derived a unique linear combination of the four Doppler data that is unaffected by the troposphere, ionosphere, and mechanical vibrations of the ground antenna, and allows the experimenters to further reduce by several orders of magnitude, at selected Fourier components, the frequency fluctuations generated by the onboard clock. This linear combination is optimal in that it minimizes the magnitude of the frequency fluctuations of the remaining noise sources. The corresponding root-mean-squared (r.m.s.) noise level was estimated to be equal to 4.7×10^{-18} at 10^{-3} Hz in the assumption of calibrating the frequency fluctuations induced by the interplanetary plasma, and integrating for a period of forty days.

In this paper we extend the theoretical framework derived in Paper 1 to the configuration of two spacecraft tracking each others with coherent laser light. The main result of this paper is that the frequency fluctuations due to the lasers, which are the main noise sources in these tracking experiments, can be reduced by several orders of magnitude at selected Fourier components. This makes spacecraft to spacecraft coherent laser tracking the equivalent of a *xylophone interferometer* detector of gravitational waves. Interferometric measurements of gravitational radiation with only two free-falling particles are possible because the transfer functions of a gravitational wave pulse and the frequency fluctuations of the lasers to the tracking observables, are different when the wavelength of the gravitational wave is shorter than the distance between the spacecraft [5, 9, 10].

In recent years there has been increased interest from NASA to explore the solar system with multi-spacecraft missions. If adequate optical payloads could be added to these missions, searches for gravitational waves through spacecraft to spacecraft laser tracking could be performed.

The data analysis technique presented in this paper could also be implemented with the future LISA mission [3] as a backup option in case of failure of one of the three spacecraft. This space-based, one-bounce Michelson interferometer detector of gravitational waves, has been considered by the European

Space Agency (ESA) and the National Aeronautics and Space Administration (NASA) to be launched during the first decade of next century. With its three spacecraft flying in a triangular formation and simultaneously tracking each others with coherent laser light, LISA can simultaneously generate data from three Michelson interferometers. In the eventuality of losing one spacecraft, two of the three coherent links between the spacecraft are lost, making impossible the calibration via Michelson interferometry of the frequency fluctuations due to the master laser. The implementation of the method described in this paper, however, would still allow the experimenters to make narrow-band measurements of gravitational waves with the remaining two spacecraft. In what follows we present an outline of this paper.

In Section II we give a summary of the main results derived in Paper 1, which provides the theoretical framework for this paper. After deriving the transfer functions of the noise sources affecting the one-way and two-way tracking data sets, we show that there exist two possible linear combinations of the two one-way Doppler data that allow the reduction of the noise from the lasers at selected Fourier frequencies. These are what we will refer to as the frequencies of the xylophone.

Since the xylophone frequencies are equal to multiple integers of the inverse of the round trip light time, any variation in the distance between the spacecraft implies changes in these frequencies. However, for selected trajectories of the two spacecraft, we can successfully implement the xylophone technique by integrating the data for time intervals during which the variations of the xylophone frequencies are smaller than the frequency resolution bin. Examples of trajectories fulfilling this requirement are presented in Section III.

The strain sensitivities achievable with this technique, when implemented with two spacecraft having the same instrumentation as the one utilized by LISA [3], are presented in Section IV. We find that a strain sensitivity of 1.9×10^{-22} at the frequency 3×10^{-2} Hz can be achieved when searching for sinusoids and by integrating the data for three weeks. At this sensitivity level, gravitational radiation from galactic binary systems should be observable.

Sensitivities to gravitational wave bursts and a stochastic background of gravitational radiation are also presented in this Section, while in Section V we provide our comments and conclusions.

II Spacecraft to Spacecraft Coherent Laser Tracking as a Xylophone Interferometer

Let us consider two spacecraft in their interplanetary trajectories, each acting as a free falling test particle, and continuously tracking each others via coherent laser light. One spacecraft, which we will refer to as spacecraft a, transmits a laser beam of nominal frequency ν_0 to the other spacecraft (spacecraft b). The phase of the light received at spacecraft b is used by a laser onboard spacecraft b for coherent transmission back to spacecraft a. The relative two-way frequency (or phase) changes $\Delta\nu/\nu_0$, as functions of time, are then measured at a photo detector [3]. When a gravitational wave crossing the solar system propagates through this electromagnetic link, it causes small perturbations in $\Delta\nu/\nu_0$, which are replicated three times in the Doppler data with maximum spacing given by the two-way light propagation time between the two spacecraft [5].

Let us introduce a set of Cartesian orthogonal coordinates (X, Y, Z) centered on one of the two spacecraft, say spacecraft a. The Z axis is assumed to be oriented along the direction of propagation of a gravitational wave pulse, and (X, Y) are two orthogonal axes in the plane of the wave (see Figure 1). In this coordinate system we can write the two-way Doppler response, measured by spacecraft a at time t, as follows [5]

$$\left(\frac{\Delta\nu(t)}{\nu_0}\right)_a \equiv y_{2a}(t) = -\frac{(1-\mu)}{2}\, h(t) \; - \; \mu\, h(t-(1+\mu)L) + \frac{(1+\mu)}{2}\, h(t-2L)$$
$$+ C_a(t-2L) \; - \; C_a(t) \; + \; 2B_b(t-L) \; + \; B_a(t-2L) \; + \; B_a(t)$$
$$+ TR_b(t-L) \; + \; N_{2a}(t) \; , \tag{1}$$

where $h(t)$ is equal to

$$h(t) = h_+(t)\cos(2\phi) + h_\times(t)\sin(2\phi) \; . \tag{2}$$

Here $h_+(t)$, $h_\times(t)$ are the wave's two amplitudes with respect to the (X, Y) axis, (θ, ϕ) are the polar angles describing the location of spacecraft b with respect to the (X, Y, Z) coordinates, μ is equal to $\cos\theta$, and L is the distance between the two spacecraft (units in which the speed of light $c = 1$).

We have denoted with $C_a(t)$ the random process associated with the frequency fluctuations of the laser onboard spacecraft a; $B_a(t)$, $B_b(t)$ are the joint effects of the noises from buffeting by non gravitational forces on the test masses [3] onboard spacecraft a and b respectively, $TR_b(t)$ is the noise due to the optical transponder on board spacecraft b, and $N_{2a}(t)$ is the noise from the photo detector on board spacecraft a where two-way phase changes are measured [3].

From Eq. (1) we deduce that gravitational wave pulses of duration longer than the round trip light time $2L$ have a frequency response $y_{2a}(t)$ that, to first order, tends to zero. The tracking system essentially acts as a pass-band device, in which the low-frequency limit f_l is roughly equal to $(2L)^{-1}$ Hz, and the high-frequency limit f_H is set by the shot noise at the photo detector.

In Eq. (1) it is also important to note the characteristic time signatures of the random processes $C_a(t)$, $B_a(t)$, and $B_b(t)$. The time signature of the noise $C_a(t)$ can be understood by observing that the frequency of the signal received at time t contains laser frequency fluctuations transmitted $2L$ seconds earlier. By subtracting from the frequency of the received signal the frequency of the signal transmitted at time t, we also subtract the frequency fluctuations $C_a(t)$ with the net result shown in Eq. (1) [4, 5, 9]. As far as the fluctuations due to buffeting of the test-mass onboard spacecraft a are concerned, the frequency of the received signal is affected at the moment of reception as well as $2L$ seconds earlier. Since the frequency of the signal generated at time t does not contain yet any of these fluctuations, we conclude that $B_a(t)$ is positive-correlated at the round trip light time $2L$ [4, 5, 9]. The time signature of the noise $B_b(t)$ in Eq. (1) can be understood through similar considerations.

Besides two-way measurements, one-way coherent laser tracking can also be symmetrically recorded by both spacecraft [7, 9, 11] (see Figure 2). If we assume the time keeping systems on board the two spacecraft to be synchronized, denote with $\overline{\nu}_0$ the frequency of the signal transmitted by spacecraft b, with $y_{1a}(t)$ the one-way data measured at time t on board spacecraft a, with $y_{1b}(t)$ and $y_{2b}(t)$ respectively the one-way and two-way measurements performed on board spacecraft b at the same time t, then a gravitational wave pulse appears in the data $y_{1a}(t)$, $y_{1b}(t)$, and $y_{2b}(t)$, with the following signatures [9, 12]

$$y_{1a}(t) = \frac{(1-\mu)}{2} \; [h(t-(1+\mu)L) \; - \; h(t)] \; + \; C_b(t-L) \; - \; C_a(t)$$

$$+ \; B_a(t) \; + \; B_b(t-L) \; + \; N_{1a}(t) \; , \tag{3}$$

$$y_{1b}(t) = \frac{(1+\mu)}{2}\ [h(t-L)\ -\ h(t-\mu L)]\ +\ C_a(t-L)\ -\ C_b(t)$$

$$+\ B_a(t-L)\ +\ B_b(t)\ +\ N_{1b}(t)\ , \qquad (4)$$

$$y_{2b}(t) = -\frac{(1+\mu)}{2}\ h(t-\mu L)\ +\ \mu\ h(t-L) + \frac{(1-\mu)}{2}\ h(t-2L-\mu L)$$

$$+\ C_b(t-2L)\ -\ C_b(t)\ +\ B_b(t-2L)\ +\ B_b(t)\ +\ 2B_a(t-L)$$

$$+\ TR_a(t-L)\ +\ N_{2b}(t)\ . \qquad (5)$$

In Eqs. (3, 4, 5) we have denoted by $C_b(t)$ the random process associated with the frequency fluctuations of the laser onboard spacecraft b; $TR_a(t)$ is the noise due to the optical transponder on board spacecraft a, and $N_{1a}(t)$, $N_{1b}(t)$, $N_{2b}(t)$ are the shot noises in the one-way and two-way data. The data $y_{1a}(t)$, $y_{2a}(t)$, $y_{1b}(t)$, $y_{2b}(t)$ are then recorded, time tagged, and telemetered back to Earth for analysis at a later time during the mission.

Among all the noise sources included in Eqs. (1, 3, 4, 5) the frequency fluctuations due to the lasers are expected to be the largest. A space-qualified single-mode laser, such as a diode-pumped Nd:YAG ring laser of frequency $\nu_0 = 3.0 \times 10^{14}$ Hz and phase-locked to a Fabry-Perot optical cavity, is expected to have a spectral level of frequency fluctuations equal to about 1.0 $\times 10^{-13}/\sqrt{Hz}$ [3] in the millihertz band. Frequency stability measurements performed on such a laser by McNamara *et al.* [13] indicate that a stability of about 1.0 $\times$ $10^{-14}/\sqrt{Hz}$ might be achievable in the same frequency band.

For the moment we will not make any assumptions on the frequency stability of the onboard lasers, and return to this point later. We will focus instead on their transfer functions and on the transfer function of the gravitational wave signal as shown in Eqs. (1, 3, 4 5). If we denote with $\widetilde{y_{1a}}(f)$ the Fourier transform of the time series $y_{1a}(t)$, this is equal to

$$\widetilde{y_{1a}}(f) \equiv \int_{-\infty}^{+\infty} y_{1a}(t)\ e^{2\pi i f t}\ dt\ . \qquad (6)$$

Similarly we will denote with $\widetilde{y_{2a}}(f)$, $\widetilde{y_{1b}}(f)$, $\widetilde{y_{2b}}(f)$ the Fourier transforms of $y_{2a}(t)$, $y_{1b}(t)$, $y_{2b}(t)$ respectively.

By taking the Fourier transform of Eq. 1 for instance, we derive the following expression

$$\widetilde{y_{2a}}(f) = \left[\frac{(\mu - 1)}{2} - \mu\, e^{2\pi i f(1+\mu)L} + \frac{(1+\mu)}{2}\, e^{4\pi i f L}\right] \widetilde{h}(f)$$

$$+ \widetilde{C_a}(f)\, \left[e^{4\pi i f L} - 1\right] + \widetilde{B_a}(f)\, \left[e^{4\pi i f L} + 1\right] + 2\, \widetilde{B_b}(f)\, e^{2\pi i f L}$$

$$+ \widetilde{TR_b}(f)\, e^{2\pi i f L} + \widetilde{N_a}(f)\ , \qquad (7)$$

Note that the transfer function of the noise C_a is equal to zero at frequencies that are multiple integers of the inverse of the round-trip-light time, while the transfer function of the gravitational wave signal is in general different from zero. By making coherent laser tracking measurements at these frequencies, we are in fact making *xylophone interferometric measurements* of gravitational waves.

We have used the two-way response measured on spacecraft a as an example for pointing out that a narrow-band interferometer can be implemented with only two test particles. It is easy to see, however, that $y_{2a}(t)$ is not the only data set that allows us to remove the frequency fluctuations of the lasers. In Paper 1 we have shown that, among the noise sources C_a, C_b, B_a, B_b, only one of them can be removed at any time t by properly combining the four data sets. This result follows from the fact that the two-way responses can be written as linear combinations of the two one-way responses in the following way

$$y_{2a}(t) - y_{1a}(t) - y_{1b}(t - L) = TR_b(t - L) + N_{2a}(t) - N_{1a}(t) - N_{1b}(t - L)\ , \qquad (8)$$

$$y_{2b}(t) - y_{1b}(t) - y_{1a}(t - L) = TR_a(t - L) + N_{2b}(t) - N_{1b}(t) - N_{1a}(t - L)\ . \qquad (9)$$

Eqs. (8, 9) imply that we can regard the one-way data as primary data sets, since they contain all the information about the gravitational wave signal and the noise sources $C_a(t)$, $C_b(t)$, $B_a(t)$, $B_b(t)$. Furthermore, by comparing the two-way data against the *synthesized* two-way data given by Eqs. (8, 9), we can assess the overall performance of the transponders and the photo detectors. In other words we can take advantage of the data redundancy by validating the quality of the two-way data.

Since the fluctuations due to the lasers are the dominant noise sources in the tracking responses, and only one of them can be removed at any time t [9], from Eqs. (8, 9) it follows that there are only two linear combinations of the two one-way data that do not contain one of the random processes $C_a(t)$, $C_b(t)$. In the Fourier domain they have the following form

$$\widetilde{x}(f) \equiv \widetilde{y_{1a}}(f) \ + \ \widetilde{y_{1b}}(f) \ e^{2\pi i f L} \ , \tag{10}$$

$$\widetilde{y}(f) \equiv \widetilde{y_{1b}}(f) \ + \ \widetilde{y_{1a}}(f) \ e^{2\pi i f L} \ . \tag{11}$$

It is easy to verify that the transfer functions of the noises C_a, C_b in Eqs. (10, 11) are null at frequencies that are multiple integers of the inverse of the round-trip-light time $2L$. The two-way data given by Eqs. (10, 11) are not the only two linear combinations of the one-way data whose lasers' transfer functions have nulls. This property is also fulfilled by the following two linear combinations of the two one-way data sets

$$y_+(t) \equiv \frac{1}{2} \ [y_{1a}(t) \ + \ y_{1b}(t)] \ , \tag{12}$$

$$y_-(t) \equiv \frac{1}{2} \ [y_{1a}(t) \ - \ y_{1b}(t)] \ , \tag{13}$$

where we have included a factor 1/2 in order to normalize to one the maximum of the antenna pattern of the gravitational wave signal (see Eqs. (16, 17) below). The two linear combinations $y_+(t)$, $y_-(t)$ are invariant (modulo a sign change) under permutation of the spacecraft indices a, b, and knowledge of the distance L between the spacecraft is not required in order to construct them, contrary to what happens with $x(t)$ and $y(t)$.

If we take into account Eqs. (3, 4), the Fourier transforms of Eqs. (12, 13) assume the following forms

$$\widetilde{y_+}(f) = H_+(\mu, f) \ \widetilde{h}(f) \ + \ \frac{1}{2} \ \left[\widetilde{C_a}(f) + \widetilde{C_b}(f)\right] \ (e^{2\pi i f L} - 1) \ + \ \frac{1}{2} \ \left[\widetilde{N_{1a}}(f) + \widetilde{N_{1b}}(f)\right]$$
$$+ \frac{1}{2} \ \left[\widetilde{B_a}(f) + \widetilde{B_b}(f)\right] \ (e^{2\pi i f L} + 1) \ , \tag{14}$$

$$\widetilde{y_-}(f) = H_-(\mu, f) \ \widetilde{h}(f) \ + \ \frac{1}{2} \ \left[\widetilde{C_a}(f) - \widetilde{C_b}(f)\right] \ (e^{2\pi i f L} + 1) \ + \ \frac{1}{2} \ \left[\widetilde{N_{1a}}(f) - \widetilde{N_{1b}}(f)\right]$$
$$+ \frac{1}{2} \ \left[\widetilde{B_a}(f) - \widetilde{B_b}(f)\right] \ (e^{2\pi i f L} - 1) \ , \tag{15}$$

where we have denoted with $H_+(\mu, f)$, $H_-(\mu, f)$ the following antenna patterns of the responses $\widetilde{y_+}(f)$, $\widetilde{y_-}(f)$ respectively

$$H_+(\mu, f) = \frac{1}{4} \left[(\mu - 1)\ (1 - e^{2\pi i(\mu+1)fL}) \ + \ (\mu + 1)\ (1 - e^{2\pi i(\mu-1)fL}) e^{2\pi i fL}\right] , \tag{16}$$

$$H_-(\mu, f) = \frac{1}{4} \left[(\mu - 1)\ (1 - e^{2\pi i(\mu+1)fL}) \ - \ (\mu + 1)\ (1 - e^{2\pi i(\mu-1)fL}) e^{2\pi i fL}\right] . \tag{17}$$

Note that the transfer functions of the noises due to the lasers are null at frequencies that are even and odd multiple integers of the inverse of the round-trip-light-time $2L$ respectively

$$f_+(k) \equiv \frac{2k}{2L} \equiv f_0\ 2k$$

$$f_-(k) \equiv \frac{2k-1}{2L} \equiv f_0\ (2k - 1)\ ; \qquad k = 1, 2, 3, ... , \tag{18}$$

where f_0 is equal to the inverse of the round-trip-light time. If we define Δf to be the frequency resolution of our data set (equal to the inverse of the integration time τ), to first order in $(\Delta f\ L)$ and at the frequencies $f_+(k)$, $f_-(k)$, the responses $\widetilde{y_+}(f_+)$, $\widetilde{y_-}(f_-)$, can be approximated by the following expressions [9, 14]

$$\widetilde{y_+}(k) = \frac{1}{2}\ \mu\ [1 - e^{2\pi i k\mu})]\ \tilde{h}(f_+)\ \pm\ \frac{1}{2}\ \left[\widetilde{C_a}(f_+) + \widetilde{C_b}(f_+)\right]\ (\pi i \Delta f L)$$
$$+ \left[\widetilde{B_a}(f_+) + \widetilde{B_b}(f_+)\right]\ +\ \frac{1}{2}\ \left[\widetilde{N_{1a}}(f_+) + \widetilde{N_{1b}}(f_+)\right] , \tag{19}$$

$$\widetilde{y_-}(k) = \frac{1}{2}\ \mu\ \left[1 + e^{\pi i(2k-1)\mu}\right]\ \tilde{h}(f_-)\ \pm\ \frac{1}{2}\ \left[\widetilde{C_a}(f_-) - \widetilde{C_b}(f_-)\right]\ (\pi i \Delta f L)$$
$$+ \left[\widetilde{B_b}(f_-) - \widetilde{B_1}(f_-)\right]\ +\ \frac{1}{2}\ \left[\widetilde{N_{1a}}(f_-) - \widetilde{N_{1b}}(f_-)\right] \quad k = 1, 2, 3, ... \tag{20}$$

If we consider two spacecraft separated by a distance $L = 5\ \times 10^6$ km on the same circular heliocentric orbit, and assume an integration time τ of three months as a numerical example, we find that the amplitudes of the frequency fluctuations due to the lasers are reduced at the xylophone frequencies by a factor of

$$\frac{\pi\ \Delta f\ L}{c} = 6.5 \times 10^{-6} . \tag{21}$$

Eqs. (19, 20) show some interesting, and somewhat peculiar, properties of the remaining gravitational wave signals at the xylophone frequencies. The response to a gravitational wave pulse goes to zero not only when the wave propagates along the line of site between the spacecraft ($\mu = \pm 1$), but also for directions orthogonal to it ($\mu = 0$). This is consequence of the fact that for $\mu = 0$ the Doppler responses y_+, y_- to a gravitational wave become *two-pulses*, identical to the responses of the laser noises, and therefore they cancel out at the xylophone frequencies.

For sources randomly distributed in the sky, as in the case of a stochastic background of gravitational waves, we can assume the angles (θ, ϕ) to be random variables uniformly distributed over the sphere. Since the average over (θ, ϕ) of the responses given in Eqs. (14, 15) are equal to zero, we find that their variances (denoted with $\Sigma^2_+(f)$, $\Sigma^2_-(f)$) are equal to

$$\Sigma^2_-(f) = \left[\frac{1}{3} - \frac{1}{2\left(\frac{\pi f}{f_0}\right)^2}\right] + \left[\frac{1}{6} + \frac{1}{2\left(\frac{\pi f}{f_0}\right)^2}\right] \cos\left(\frac{\pi f}{f_0}\right) - \left[\frac{\cos^2\left(\frac{\pi f}{2f_0}\right)}{\left(\frac{\pi f}{f_0}\right)^3}\right] \sin\left(\frac{\pi f}{f_0}\right) , \tag{22}$$

$$\Sigma^2_+(f) = \left[\frac{1}{3} - \frac{1}{2\left(\frac{\pi f}{f_0}\right)^2}\right] - \left[\frac{1}{6} + \frac{1}{2\left(\frac{\pi f}{f_0}\right)^2}\right] \cos\left(\frac{\pi f}{f_0}\right) + \left[\frac{\sin^2\left(\frac{\pi f}{2f_0}\right)}{\left(\frac{\pi f}{f_0}\right)^3}\right] \sin\left(\frac{\pi f}{f_0}\right) , \tag{23}$$

At the xylophone frequencies f_-, f_+, the functions Σ^2_-, Σ^2_+, are equal to the following monotonically increasing functions of the integer k

$$\Sigma^2_+(k) = \frac{1}{6} - \frac{1}{4\pi^2 k^2} , \tag{24}$$

$$\Sigma^2_-(k) = \frac{1}{6} - \frac{1}{\pi^2(2k-1)^2} \quad ; \quad k = 1, 2, 3, \ldots . \tag{25}$$

III Estimated Sensitivities

In order to take advantage of the xylophone technique it is necessary to integrate over a sufficiently long period of time. This is because we want to reduce the noise due to the lasers to a level as close as possible to that identified by the remaining noise sources at the xylophone frequencies (see Eqs. (19, 20)). Since the xylophone frequencies change in time as the distance between the spacecraft varies, we can not coherently integrate our data indefinitely. Coherent integration can be performed only on a time scale τ during which the variations of the xylophone frequencies are smaller than the frequency resolution $\Delta f = 1/\tau$.

As an example of a xylophone interferometer detector of gravitational waves, let us consider two spacecraft separated by an average distance of 5.0×10^6 km on the same heliocentric orbit. This orbital configuration can be achieved by launching two spacecraft with a rocket capable of injecting them into the same Earth escape trajectory. Subsequently the two probes will drift away from the Earth, and will start reducing their speeds, using their own propulsion systems, at pre-determined moments. This maneuver will take them into their final orbit, in which they will follow the Earth around the Sun in the plane of the ecliptic. We point out that this sequence of maneuvers is a subset of a more complicated sequence needed for positioning the LISA interferometer in its final operational configuration [3], and it is therefore expected to be realizable. LISA will have its three spacecraft flying in a circular orbit inclined 60^0 with respect to the plane of the ecliptic, and following the Earth approximately 20^0 behind it as seen from the Sun.

In order to identify the maximum time of coherent integration for our xylophone interferometer detector, we need to derive the distance $L(t)$ between the spacecraft as a function of time. $L(t)$ can be obtained from the coordinates of the two spacecraft relative to a coordinate system centered on the Sun. These can be written in the following parametric form [15]

$$\begin{aligned} x_a(\xi) &= \alpha\ (\cos(\xi) - e) \qquad & y_a(\xi) &= \alpha\ \sqrt{1-e^2}\ \sin(\xi) \\ x_b(\xi) &= \alpha\ (\cos(\xi + \bar{\xi}) - e) \qquad & y_b(\xi) &= \alpha\ \sqrt{1-e^2}\ \sin(\xi + \bar{\xi}) \\ t &= \frac{T}{2\ \pi}\ (\xi - e\ \sin\xi)\ , \end{aligned} \tag{26}$$

where $t = 0$ has been chosen to coincide with the instant when spacecraft a is at perihelion. In Eq. (26) T is the period of the orbit (one year), ξ is the parameter describing the trajectory, α is the semi-major axis of the ellipse, $e = 0.017$ is the eccentricity of the trajectory (equal to the eccentricity of the Earth's trajectory), and $\bar{\xi} = 1.9^0$ is the value of the parameter ξ corresponding to a separation of 5.0×10^6 km between the two spacecraft at $t = 0$. From Eq. (26) it is easy to derive the following parametric equation of the distance $L(\xi)$ between the probes

$$L^2(\xi) = L^2(0) + \alpha^2\ e^2\ \left\{\sin^2(\bar{\xi}) - \left[\sin(\bar{\xi} + \xi) - \sin(\xi)\right]^2\right\}\ . \tag{27}$$

As a consequence of the magnitude of the geometrical quantities e, $L(0)$, $\overline{\xi}$, and α entering into Eq. (27), we can rewrite $L(\xi)$ in the following approximated form

$$L(\xi) \approx L(0) \left[1 + \frac{\alpha^2 \ e^2 \ \overline{\xi}^2}{2 \ L^2(0)} \ \sin^2(\xi)\right] \ . \tag{28}$$

From the definition of the frequencies f_-, f_+, we can derive the relationship between the variation of the xylophone frequencies, δf_-, δf_+, and the relative change in the distance between the spacecraft, $\delta L(\xi)/L(0)$. Since, for instance, δf_- is related to $\delta L(\xi)$ by the following equation

$$\delta f_- = f_0 \ \times (2k - 1) \ \times \frac{\delta L(\xi)}{L(0)} \ , \tag{29}$$

by requiring it to be smaller than the frequency resolution $\Delta f = 1/\tau$, we obtain from Eqs. (26, 28, 29) the following inequality

$$f_0 \ \times (2k - 1) \ \times \frac{\alpha^2 \ e^2 \ \overline{\xi}^2}{2 \ L^2(0)} \ \sin^2(\xi_0) \le \frac{2 \ \pi}{T \ (\xi_0 - e \ \sin(\xi_0))} \ , \tag{30}$$

where ξ_0 corresponds to $t = \tau$ in Eq. (26). Eq. (30) can be solved numerically in terms of the parameter ξ_0, for a given choice of the integer k. If we assume $k = 1$, that is to say we consider the variation of the fundamental frequency $f_0 = 3 \times 10^{-2}$ Hz, we find that we can integrate coherently our data for as long as 21 days. If we require instead all the xylophone frequencies to change by less than the frequency resolution, we need to calculate the maximum integration time for the value of k corresponding to the largest xylophone frequency we want to include in our observable frequency band. With $k = 25$, which implies $f_-(k = 25) = 1.5$ Hz, we find a corresponding maximum integration time of 5.5 days.

Let us turn now to the trajectory of the spacecraft in the LISA mission. It has been calculated by Folkner *et al.* [16] that the relative longitudinal speeds between the three pairs of spacecraft, during approximately the first year of the mission, can be written in the following approximated form

$$V_{i,j}(t) = V_{i,j}^{(0)} \ \sin\left(\frac{2\pi t}{T_{i,j}}\right) \qquad (i,j) = (1,2) \ ; \ (1,3) \ ; \ (2,3) \ , \tag{31}$$

where we have denoted with $(1,2), (1,3), (2,3)$ the three possible spacecraft pairs, $V_{i,j}^{(0)}$ are constant velocities, and $T_{i,j}$ are the periods for the pairs (i, j). In reference [16] it has also been shown that the LISA trajectory can be selected in such a way that two of the three arms' rates of change are essentially equal during the first year of the mission. This configuration is particularly attractive because it implies an almost null variation in differential armlength for one of the three interferometers. Following reference [16], we will assume $V^{(0)}_{1,2} = V^{(0)}_{1,3} \neq V^{(0)}_{2,3}$, with $V^{(0)}_{1,2} = 1$ m/s, $V^{(0)}_{2,3} = 13$ m/s, $T_{1,2} = T_{1,3} \approx 4$ months, and $T_{2,3} \approx 1$ year. With these numerical values we can calculate the maximum integration times for different xylophone frequencies. The calculation is similar to the one we have performed in our example-trajectory above, and therefore we will not go through it here. The results of this analysis, however, indicate that the data from the two pairs of spacecraft, $(1, 2), (1, 3)$, can be integrated coherently at the frequency $f_0 = 3 \times 10^{-2}$ Hz for about 10 days. A shorter integration time of about 3 days is needed instead to make xylophone measurements at the frequency 1.5 Hz. For the remaining pair of spacecraft, due to their larger relative speed, we have found that coherent integration at f_0 can be performed for about 6 days, while at 1.5 Hz the maximum integration time goes down to about 2 days.

The numerical values of the maximum integration times derived above allow us to estimate, at the xylophone frequencies, the one-sided power spectral densities of the noises affecting the two data sets y_-, y_+. In what follows we will consider two spacecraft with identical optical and mechanical payloads, and we will assume them to be equal to those that will fly onboard the LISA spacecraft [3]. We will also assume the random processes associated with the noise sources affecting the stability of the coherent one-way tracking data to be uncorrelated with each others, and their one-sided power spectral densities to be consistent with those given in reference [3]. Since our xylophone will be sensitive to gravitational radiation at frequencies equal to or larger than the inverse of the round-trip-light-time ($c/2L = 3 \times 10^{-2}$ Hz), the dominant noise sources determining its strain sensitivity will be the photon-shot noises, and the frequency fluctuations of the lasers [3].

After taking into account Eqs. (14, 15), and the expressions of the one-sided power spectral densities for the shot-noise and the frequency fluctuations of the laser given in reference [3], the one-sided power spectral densities $S_{y_-}(f)$, $S_{y_+}(f)$ of the noises in the responses y_-, y_+ respectively, estimated in the frequency band of the xylophone, can be written as follows

$$S_{y_-}(f) = 10^{-38}\ f^2\ +\ \left[10^{-28}\ f^{-2/3}\ +\ 6.3\ \times 10^{-37}\ f^{-3.4}\right]\ \cos^2\left(\frac{\pi f}{2 f_0}\right)\ , \tag{32}$$

$$S_{y_+}(f) = 10^{-38}\, f^2 \; + \; \left[10^{-28}\, f^{-2/3} \; + \; 6.3 \; \times 10^{-37}\, f^{-3.4}\right] \, \sin^2\left(\frac{\pi f}{2f_0}\right) \; . \tag{33}$$

In Figure 3 (i, ii) we have plotted these two functions by assuming an integration time of 5.5 days. Note that, with such an integration time, the one-sided power spectral density of the laser noises are reduced, at the xylophone frequencies, by a factor of $(\pi\Delta f\; L)^2 = 1.2 \; \times 10^{-8}$. Since the functions $S_{y_-}(f)$, $S_{y_+}(f)$ plotted in Figure 3 are monotonically decreasing at the xylophone frequencies, we conclude that with such an integration time the noise due to the lasers is still the dominant one.

The 5.5 days time interval corresponds to a pair of spacecraft separated by a distance of 5×10^6 km in the Earth's heliocentric orbit, and implies a variation of the largest of the xylophone frequencies smaller than the frequency resolution bin. At smaller xylophone frequencies, however, the one-sided power spectral densities should be rescaled according to the appropriate maximum integration times. For instance, since at the frequency 3×10^{-2} Hz we can coherently integrate the data from these spacecraft for 21 days, the one-sided power spectral density at this frequency would be smaller than the value shown in Figure 3 (i) by a factor $(5.5/21)^2$. Similar considerations apply to the data from pairs of spacecraft in the LISA interferometer. We should remember this point when we will estimate the signal-to-noise ratios for various gravitational waveforms.

Gravitational waves can be classified into three categories, depending on their temporal behavior and time duration relative to the time of observation of a detector. A complete review of gravitational wave sources, and estimates of the corresponding strengths of interest in the frequency band of our xylophone, are given in reference [1].

Gravitational wave bursts in the millihertz frequency band could be emitted during different astrophysical scenarios. A collapse of a star cluster to form a supermassive black hole, for instance, might generate a waveform whose dominant spectral components coincide with the frequencies at which the effects of the frequency fluctuations due to the lasers are suppressed.

Another astrophysical scenario implying the emission of a gravitational wave burst is the fall of small black holes into a super massive black hole, as it might happen at the end of the merger of two galaxies each hosting a black hole at their centers. Although the temporal dependence of the gravitational wave burst radiated during the merger is unknown, the radiation emitted by the newly formed hole during the settling process can be described mathematically quite well [17,18]. The radiation in this case is strongly dominated by the black-hole quasi-normal modes, whose frequencies and damping times depend on the mass of the hole and its angular momentum. The strongest and most slowly damped of these modes is expected to be the fundamental, whose gravitational wave tensor can be written as follows [18]

$$h_{ij}(t) = \begin{cases} e_{ij}\ h_0\ e^{-(t-t_s)/\tau_0}\ \sin[2\pi f_s\ (t-t_s)], & t \geq t_s \\ 0 & t < t_s \end{cases} , \tag{34}$$

where e_{ij} is the polarization tensor, h_0 is the wave's amplitude at its time of arrival t_s, and f_s and τ_0 are the quasi-normal mode's frequency and damping time respectively. The analytic expressions for the amplitude h_0, the frequency f_s of the damped mode, and the damping time τ_0 are as follows [17, 18]

$$h_0 = \frac{1}{r}\ \left(\frac{G}{\pi^2\ c^3}\ \frac{\Delta E}{f_s}\right)^{1/2} \tag{35}$$

$$f_s = \frac{c^3}{2\pi M G}\ \left[1 - 0.63\ (1-\mathrm{a})^{0.3}\right] \tag{36}$$

$$\pi f_s \tau_0 = 2\ (1-\mathrm{a})^{-0.45} \geq 2\ . \tag{37}$$

Here r is the physical distance to the black-hole, ΔE is the energy radiated in the form of gravitational waves over the time scale $1/f_s$, a is the dimensionless rotation parameter, and M is the mass of the black-hole. When a approaches 1 we have a relativistically rotating black-hole, while $\mathrm{a} = 0$ corresponds to the radiation from a perturbed Schwarzschild hole [18]. Note that, in order for the signal to perform one oscillation before getting damped, the black-hole must rotate with an angular momentum a as large as 0.633 (the right-hand-side of Eq. (37) must be equal to π in order to have $\tau_0 = 1/f_s$).

It is well known that, when searching for bursts, the largest signal-to-noise ratio is achieved by applying matched filtering to the data [19]. The signal-to-noise ratios $SNR_-(f_s, \mathrm{a})$, $SNR_+(f_s, \mathrm{a})$, after matched filtering is applied to the Doppler data y_-, y_+, are given respectively by the following formulas [19]

$$SNR_-(f_s, \mathrm{a}) = \frac{h_0^2\ f_s^2}{2\ \pi^2}\ \int_{-\infty}^{+\infty} \frac{|H_-(\mu, f)|^2}{S_{y_-}(f)\ \left[f^4\ -\ A\ f^2\ +\ B\right]}\ df\ , \tag{38}$$

$$SNR_+(f_s, \mathrm{a}) = \frac{h_0^2\ f_s^2}{2\ \pi^2}\ \int_{-\infty}^{+\infty} \frac{|H_+(\mu, f)|^2}{S_{y_+}(f)\ \left[f^4\ -\ A\ f^2\ +\ B\right]}\ df\ , \tag{39}$$

where we have denoted with A, B the following two functions of f_s and a

$$A = 2\left[1 - \frac{(1 - \mathrm{a})^{9/10}}{16}\right] f_s^2 , \tag{40}$$

$$B = \left[1 + \frac{(1 - \mathrm{a})^{9/10}}{16}\right]^2 f_s^4 . \tag{41}$$

If we assume the energy ΔE emitted in the form of gravitational radiation to be proportional to the rest energy of the black-hole ($\Delta E = \epsilon M c^2$), by substituting Eq. (36) into Eq. (35) it is easy to derive the following expression for the wave's amplitude h_0

$$h_0 = \frac{c}{\pi r f_s} \left\{\frac{[1 - 0.63\ (1 - \mathrm{a})^{0.3}]\ \epsilon}{2\pi}\right\}^{1/2} . \tag{42}$$

Eq.(42) implies that, for a fixed angular momentum a, fractional radiated energy ϵ, and source distance r, the amplitude of the wave is inversely proportional to the center frequency, f_s. This is because the mass of the system is inversely proportional to f_s (Eq.(36)). For example, at the frequency $f_s = 3.0 \times 10^{-2}$, the corresponding mass of the Schwarzschild black hole is about $4 \times 10^5\ M_\odot$.

Owing to the spectral modulations, the signal-to-noise ratios of the filter matched to this signal will vary with center frequency, f_s. In Figure 4 (i, ii) we plot the signal-to-noise ratios for y_-, and y_+ versus f_s for non-rotating (a = 0) and highly-rotating (a = 0.99) waveforms, and for sources randomly distributed over the sky. Similar calculations can be performed for a particular direction of propagation of the signal. We should remember, however, that the averaged values of the antenna power-patterns are about a factor of two smaller than their respective maxima (see Eqs. (22, 23)).

An integration time of 5.5 days was assumed for calculating the signal-to-noise ratios plotted in Figure 4. This clearly underestimates the noise spectral levels at xylophone frequencies smaller than 1.5 Hz. For instance, since at the frequency of 3×10^{-2} Hz we can integrate for 21 days, and the signal-to-noise ratios are approximately inversely proportional to the value of the one-sided power spectral density of the noise at this frequencies, we conclude that the SNRs should be rescaled at this frequency by roughly a factor of $(21/5.5)^2$ with respect to the value given in Figure 4 (i). Note also that the SNRs in Figure 4 have been normalized to the percentage of energy radiated ϵ and the physical distance r measured in units of 1 Mpc; the explicit dependence of the SNRs on these two parameters is proportional to $\epsilon^1 r^{-2}$.

As an example application of Figure 4, consider gravitational radiation from quasi-normal mode perturbations of a black hole out to the Virgo cluster of galaxies ($\approx$ 20 Mpc). For a rapidly-rotating (a = 0.99) $9 \times 10^5 M_\odot$ black hole, the corresponding radiation frequency f_s is equal to 3×10^{-2} Hz. To achieve a matched filter SNR of 10, the quasi-normal mode vibrations of the black hole require an efficiency $\epsilon \approx 4 \times 10^{-5}$.

Let us now turn to the two remaining classes of gravitational waveforms, namely sinusoids and stochastic backgrounds of gravitational radiation. In the case of the stochastic background with bandwidth equal to center frequency, the sensitivities at the frequencies f_-, f_+, are given by the expected root-mean-squared (r.m.s.) noise levels σ_-, σ_+, of the frequency fluctuations in the bins of width f_-, f_+ respectively. These are equal to

$$\sigma_-(k) = \left[S_{y_-}(f_-(k))\ f_-(k)\right]^{1/2} \ , \tag{43}$$

$$\sigma_-(k) = \left[S_{y_+}(f_+(k))\ f_+(k)\right]^{1/2} \ , \qquad k = 1, 2, 3, \ldots. \ . \tag{44}$$

When searching for a stochastic background of gravitational radiation, the best sensitivity is achieved at the lowest xylophone frequency. At $f_-(1) = 3 \times 10^{-2}$ Hz we get an energy density per unit logarithmic frequency and per unit critical energy density Ω [1] equal to 1.4×10^{-6}, after taking into account the effect of the r.m.s. antenna pattern $\Sigma_-^2(k = 1)$ given in Eq. (25), and rescaling the value of $S_{y_-}(f_-(1))$ given in Figure 3 (i) to an integration time of 21 days.

Finally, sources of sinusoidal gravitational waves in the millihertz frequency band are expected to be inspiraling binary systems. As such a system evolves, the frequency of the emitted radiation slowly increases due to gravitational radiation reaction. If the source radiates near one of the frequencies f_-, f_+, and is sufficiently far from coalescence, then it will radiate predominantly as a sinusoid. In the millihertz band we can describe mathematically the radiation they emit quite accurately in the Newtonian approximation. In this framework, the maximum amplitude radiated by the system and the time it spends around the frequency f are given by the following expressions

$$h(t) = \frac{4\ G^{5/3}}{c^4}\ \frac{\mu}{r}\ [\pi\ f(t)\ M]^{2/3} \tag{45}$$

$$\frac{f(t)}{\dot{f}(t)} \equiv \eta(t) = \frac{5\ c^5}{96\ G}\ \frac{[\pi\ f(t)]^{-8/3}}{\mu\ (G\ M)^{2/3}} \ , \tag{46}$$

where r is the physical distance to the binary system, G is the gravitational constant, c is the speed of light, η is the characteristic time during which the binary system radiates at the frequency $f(t)$, and μ and M are the reduced and total mass of the system. In order for the signal's frequency not to change by more than the frequency resolution Δf of the data, the following condition must be satisfied

$$\dot{f}\ \tau < \frac{1}{\tau}\ . \tag{47}$$

If we divide both sides of Eq. (47) by one of the frequencies $f_-(k)$, for instance, we obtain the following relationship between the time, η_k, spent by the signal's frequency around the frequency $f_-(k)$, and the integration time τ

$$\eta_k > f_-(k)\ \times \tau^2\ . \tag{48}$$

Note that, the smaller the frequency at which we will perform our observation, the easier will be to have binary systems radiating sinusoidally during the integration time τ.

If, for instance, we assume $f = 3 \times 10^{-2}$ Hz, from Eqs. (46, 48) we derive that a binary system of two $1.4M_{\odot}$ neutron stars could be searched for by integrating the data for about 8 days before the frequency of the signal would change by more than the frequency bin. Since the sensitivities to sinusoids, σ_-, σ_+, at the xylophone frequencies f_-, f_+, can be written respectively in the following forms

$$\sigma_-(k) = \sqrt{S_{y_-}(f_-(k))\ \Delta f}\ , \tag{49}$$

$$\sigma_+(k) = \sqrt{S_{y_+}(f_+(k))\ \Delta f}\ , \tag{50}$$

we find that at 3×10^{-2} Hz the r.m.s. noise level is equal to 1.9×10^{-22}. With such a strain sensitivity, a binary system such as the one considered could be observed out to a distance of 19.5 kpc.

As a final note, we point out that an integration time of 8 days would allow also coherent integration of the data from two of the three pairs of spacecraft forming the LISA interferometer. Data from the third pair, however, could be integrated coherently only for 6 days, implying a distance of about 12.3 kpc at which such a binary systems could be observed.

IV Conclusions

We have discussed an experimental technique for performing searches of gravitational radiation in space with two spacecraft tracking each other~~s~~ via coherent laser light. The main result of our analysis, deduced in Eqs. (19, 20), shows that we can reduce, by several orders of magnitudes, the frequency fluctuations introduced in the data by the lasers. This is achieved through the two linear combinations of the two one-way data given in Eqs. (12, 13)), and by making measurements at selected Fourier components. In this respect spacecraft to spacecraft coherent laser tracking can be regarded as a *xylophone interferometer* detector of gravitational radiation.

Signal-to-noise ratios for various gravitational waveforms have been estimated in Section III. In particular, when searching for sinusoids, we have found that a strain sensitivity of 1.9×10^{-22} at the frequency 3×10^{-2} Hz can be achieved after coherently integrating the data for 21 days. At this sensitivity level, gravitational radiation from galactic coalescing binary systems should be observable.

Spacecraft to spacecraft xylophone interferometric measurements of gravitational radiation could be implemented with two spacecraft carrying an appropriate optical payload, or with the proposed broad-band, space-based laser interferometer detectors of gravitational waves.

Acknowledgements

It is a pleasure to thank Frank B. Estabrook and John W. Armstrong for several useful discussions, and their encouragement during this work. This research was performed at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

References

[1] K.S. Thorne. In: *Three hundred years of gravitation*, Eds. S.W. Hawking, and W. Israel. Cambridge University Press, Cambridge, U.K., (1987).

[2] F.B. Estabrook: *Proposal for Participation on Radio Science/Celestial Mechanics Team for an Investigation of Gravitational Radiation*, Galileo Project, JPL, (1976) (unpublished). See also F.B. Estabrook, R.W. Hellings, H.D. Wahlquist, J.D. Anderson, and R.S. Wolff. In: *Sources of Gravitational Radiation*, ed. L. Smarr (Cambridge University Press, Cambridge, 1979).

[3] LISA: (Laser Interferometer Space Antenna) *A Cornerstone Project in ESA's long term space science programme "Horison 2000 Plus"*. **MPQ 208**, (Max-Planck-Institute für Quantenoptic, Garching bei München, 1995). See also W.M. Folkner, P.L. Bender, and R.J. Stebbins *LISA mission concept study*, JPL Publication 97-16, (1998).

[4] J.W. Armstrong. In: *Gravitational Wave Data Analysis*, ed. B.F. Schutz (Dordrecht:Kluwer), p.153 (1989).

[5] F.B. Estabrook and H.D. Wahlquist, *Gen. Relativ. Gravit.* **6**, 439 (1975).

[6] J.W. Armstrong, *Radio Science*, accepted for publication (1998).

[7] R.F.C. Vessot and M.W. Levine, *Gen. Relativ. Gravit.* **10**, 181 (1979).

[8] L.L. Smarr, R.F.C. Vessot, C.A. Lundquist, R. Decher, and T. Piran, *Gen. Relativ. Gravit.* **15**, 2 (1983).

[9] M. Tinto, *Phys. Rev. D*, **53**, 5354, (1996).

[10] M. Tinto, and F.B. Estabrook, *Phys. Rev. D*, **52**, 1749, (1995).

[11] R. Hellings, G. Giampieri, M. Tinto, K. Danzmann, J. Hough, and D. Robertson, *Optics Communications*, **124**, 313, (1996).

[12] T. Piran, E. Reiter, W.G. Unruh, and R.F.C. Vessot, *Phys. Rev. D*, **34**, 984, (1986).

[13] P.W. McNamara, H. Ward, J. Hough, and D. Robertson, *Clas. Quantum Grav.*, **14**, 1543, (1997).

[14] M. Tinto, and J.W. Armstrong, *Phys. Rev. D*, submitted for publication (1998).

[15] L.D. Landau, and E. Lifzits, *Mecanique*, Editions MIR, Moscou, (1973).

[16] W.M. Folkner, F. Hechler, T.H. Sweetser, M.A. Vincent, and P.L. Bender, *Clas. Quantum Grav.*, **14**, 1543, (1997).

[17] L.P. Grishchuk, *Sov. Phys. Usp.*, **31**, 940, (1988)

[18] F. Echeverria, *Phys. Rev. D*, **40**, 3194, (1989)

[19] C. W. Helstrom, *Statistical Theory of Signal Detection*, (Pergamon, Oxford), (1968).

Figure Captions

Figure 1.

Coherent laser light of nominal frequency ν_0 is transmitted from spacecraft a to spacecraft b, and coherently transponded back. The gravitational wave train propagates along the Z direction, and the cosine of the angle between its direction of propagation and the laser beam is denoted by μ. An equivalent link from spacecraft b to spacecraft a is established. Together with two-way data, measurements of one-way coherent laser tracking can also be done by comparing at a photo detector the phase of the received signal with the phase of the local laser. See text for a complete description.

Figure 2.

Block diagram of the optical payload onboard spacecraft a, that allows the acquisition and recording of the two tracking data $y_{1a}(t)$, $y_{2a}(t)$. An identical configuration is implemented on board spacecraft b. Two lasers are needed in order to implement the four-link system. One of the two laser, referenced in the figure as Master laser, acts as primary frequency reference. The Slave laser instead is coherently locked to the phase of the incoming signal, and transmits back to the other spacecraft.

Figure 3.

The one-sided power spectral densities (i) $S_{y_-}(f)$, (ii) $S_{y_+}(f)$ of the noises entering the responses y_-, y_+ respectively. In order to calculate the minima, an integration time of 5.5 days has been assumed. See text for explanation.

Figure 4.

Signal-to-noise ratios to quasi-normal mode gravitational waves from sources randomly distributed over the sky, as a function of the normal mode frequency f_s and the angular momentum parameter a. Two values for the angular momentum, a = 0 (solid) and a = 0.99 (dashed) lines, are shown. (i) SNR_- is the signal-to-noise ratio after matched filters are applied to the data $y_-(t)$, while (ii) shows the signal-to-noise ratio SNR_+ after matched filters are applied to the data $y_+(t)$.

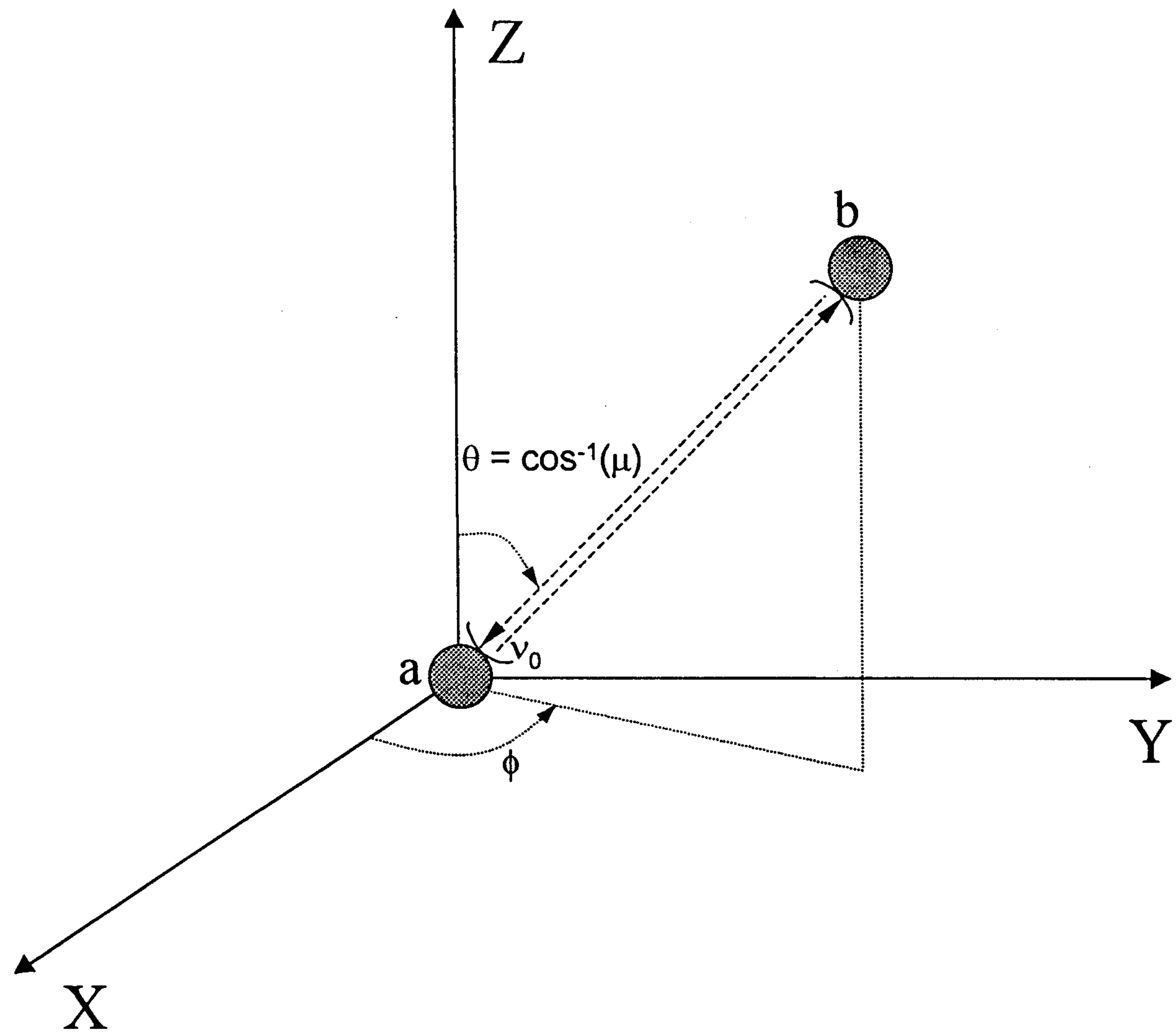

Figure 1.

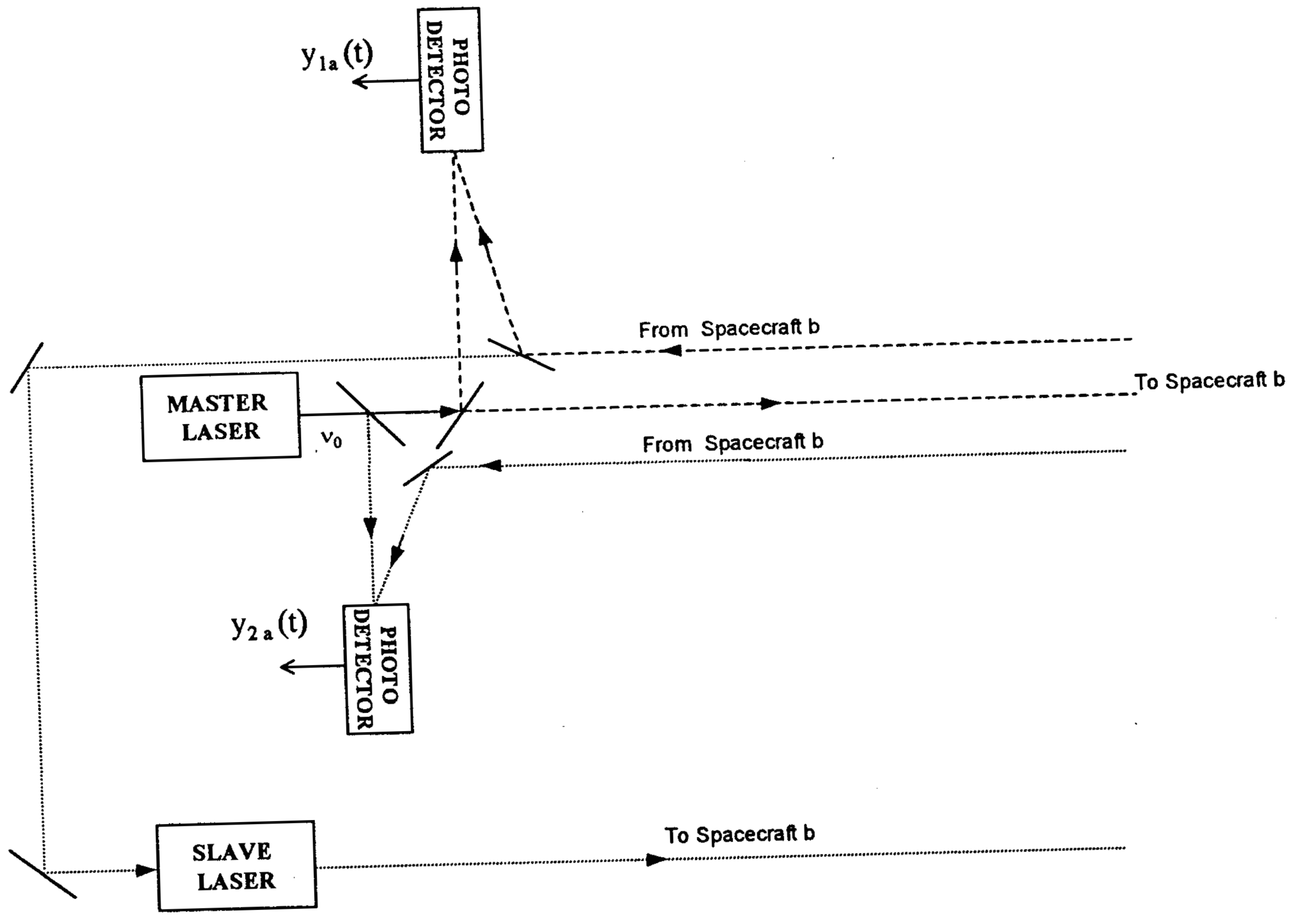
PHOTO DETECTOR
$y_{1a}(t)$
From Spacecraft b
MASTER LASER
ν_0
To Spacecraft b
From Spacecraft b
PHOTO DETECTOR
$y_{2a}(t)$
SLAVE LASER
To Spacecraft b

Figure 2.

One-Sided Power Spectral Density of the Noise

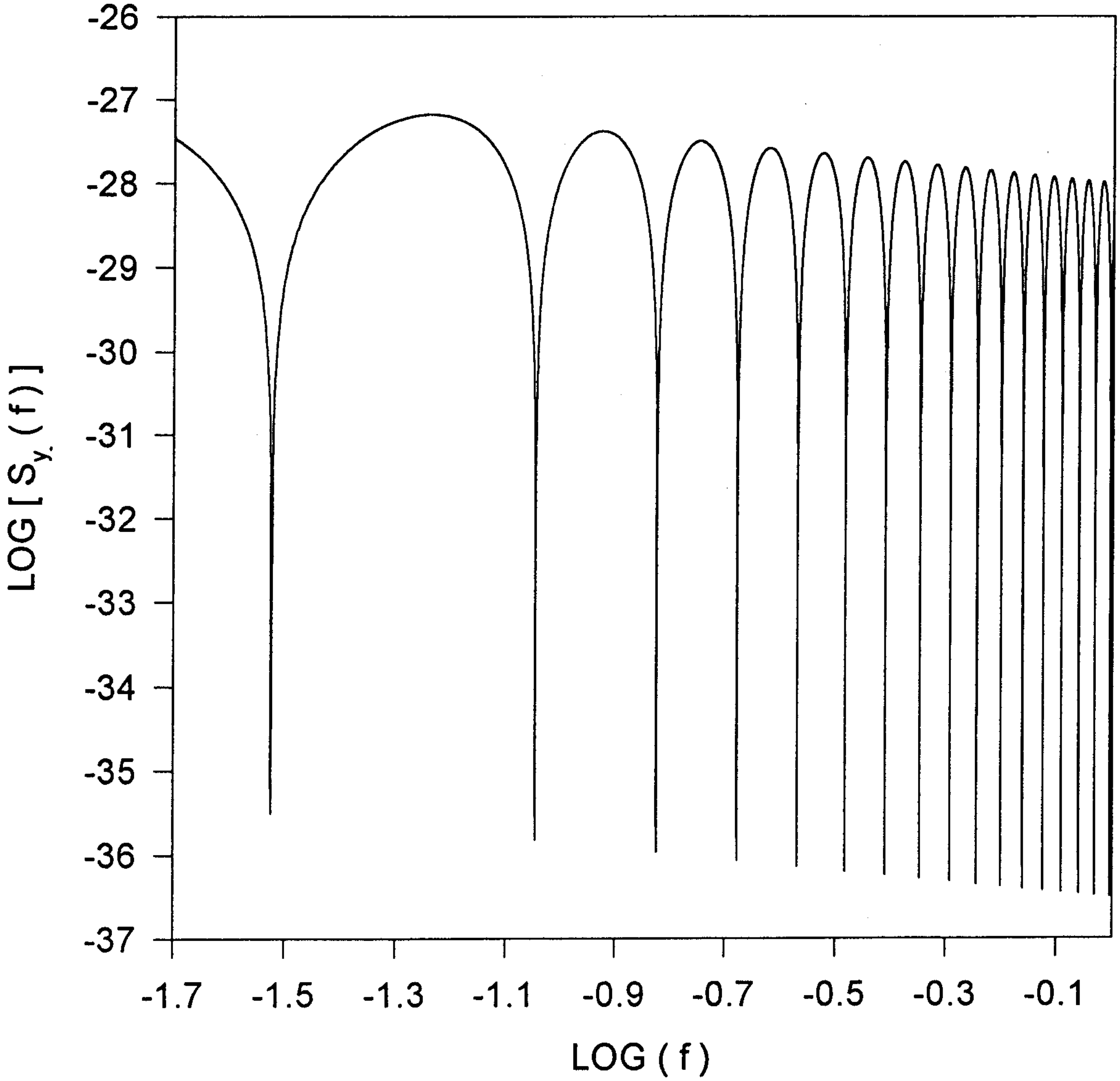

Figure 3 (i)

One-Sided Power Spectral Density of the Noise

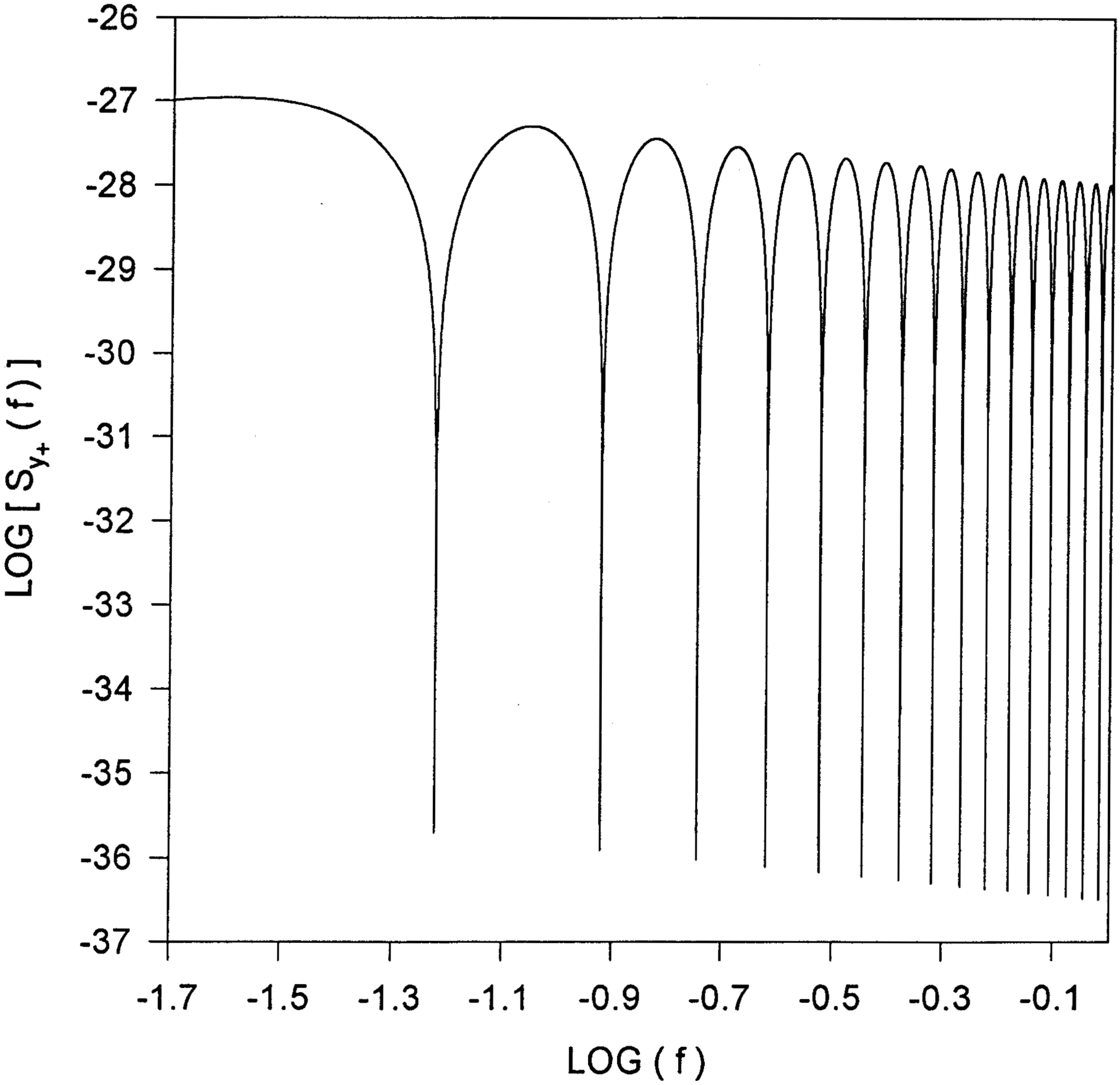

Figure 3 (ii)

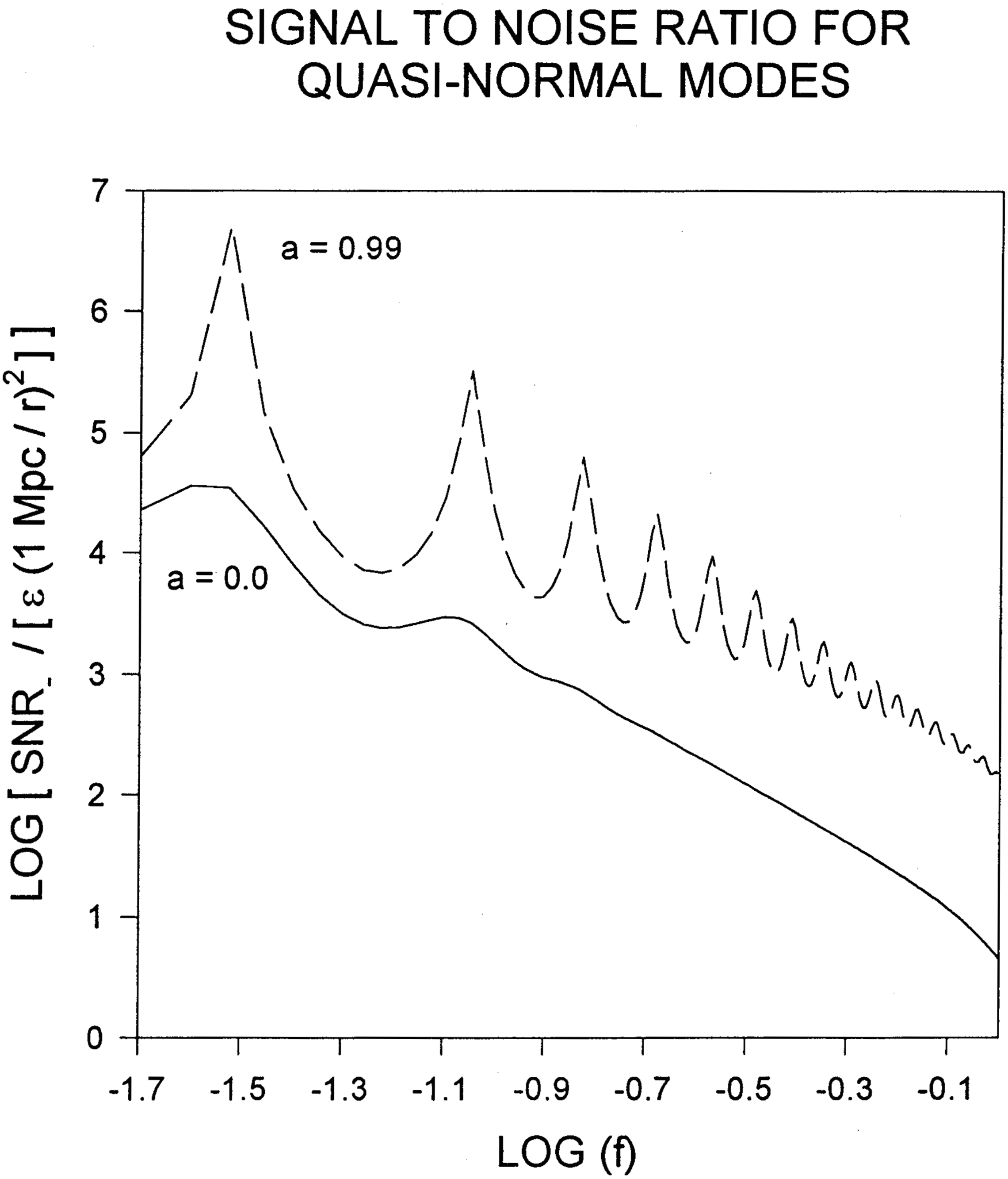
SIGNAL TO NOISE RATIO FOR
QUASI-NORMAL MODES
a = 0.99
a = 0.0
LOG [SNR_ / [ε (1 Mpc / r)^2]]
LOG (f)
0
1
2
3
4
5
6
7
-1.7
-1.5
-1.3
-1.1
-0.9
-0.7
-0.5
-0.3
-0.1

Figure 4 (i)

SIGNAL TO NOISE RATIO FOR
QUASI-NORMAL MODES

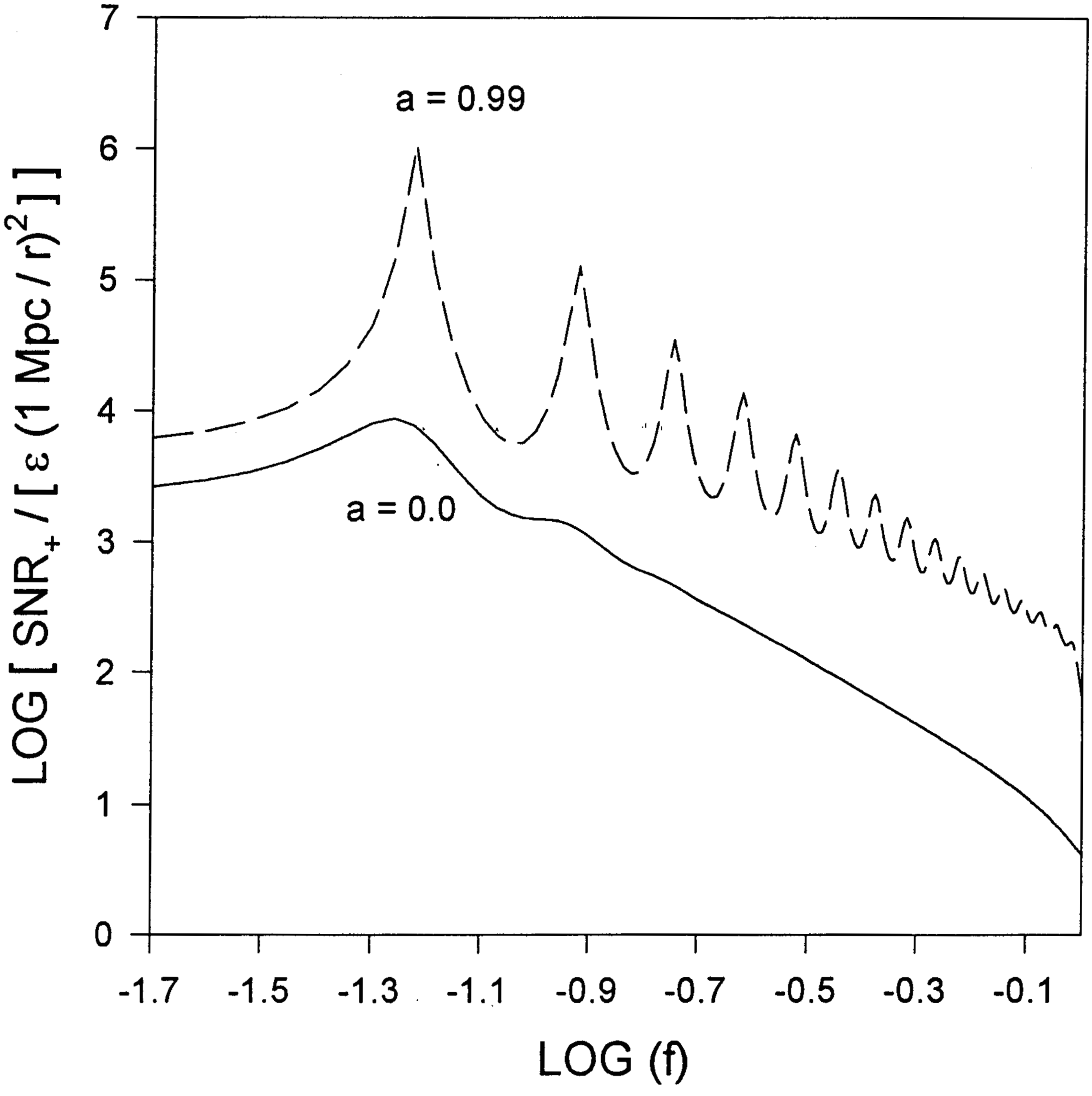

Figure 4 (ii)